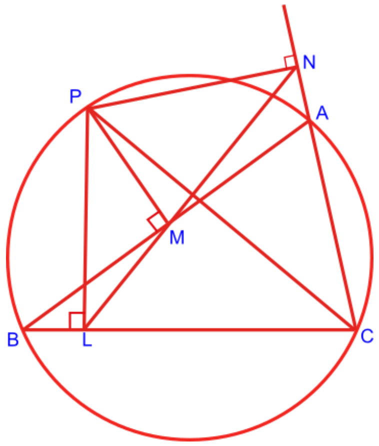
Cash Award Question of Apr-2024



In the picture, ΔABC is inscribed in a circle. P is a point on the minor arc AB. PL, PM & PN are perpendiculars dropped from P to BC, AB & CA (produced) respectively and LMN is the Simson line with respect to P.

Prove: $LN = \frac{(PC \times AB)}{D}$**, where D is the diameter of the circle.**

Question framed by
DR. M. RAJA CLIMAX
Founder Chairman,
CEOA Group of Institutions
Tamil Nadu, India

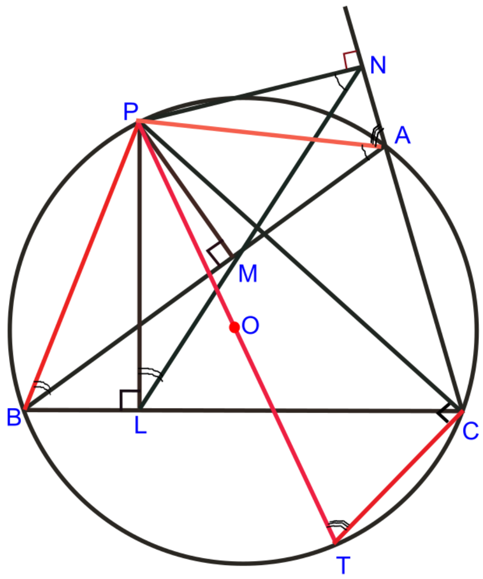
Author's Solution

Given :

ΔABC is inscribed in a circle. P is a point on the minor arc AB. PL, PM & PN are the perpendiculars drawn to BC, AB & AC respectively and LMN is the Simson line.

To Prove: $LN = \frac{(PC \times AB)}{D}$, where D is the diameter of the circle.

Construction:



Mark the centre of the circle 'O' and

draw PT, the diameter through O. Join PA, PB & TC

Proof:

$\angle PMB = \angle PLB = 90°$ (given)

$\Longrightarrow$ PBLM is concyclic

$\Longrightarrow \angle PBM = \angle PLM$ ---------------------------(1)

$\angle PMA + \angle PNA = 90° + 90° = 180°$

$\Longrightarrow PMAN$ is concyclic

$\Longrightarrow \angle PNM = \angle PAM$ ---------------------------(2)

$\Longrightarrow (1) \& (2) \rightarrow \Delta PAB \sim \Delta PNL$

$$\Longrightarrow \frac{PA}{PN} = \frac{AB}{LN}$$ -------------------------- (3)

In ΔPTC & ΔPAN

$\angle PTC = \angle PAN$ [$\because$ $Exterior\ angle = Interior\ opp.angle$]

$\angle PCT = \angle PNA = 90°$ [∠PCT borne by diameter]

$\therefore \Delta PTC \sim \Delta PAN$

$$\Longrightarrow \frac{PT}{PA} = \frac{PC}{PN}$$ --------------------------- (4)

(3) & (4) $\rightarrow LN = \frac{PC \times AB}{PT}$ -------------------------- Proved

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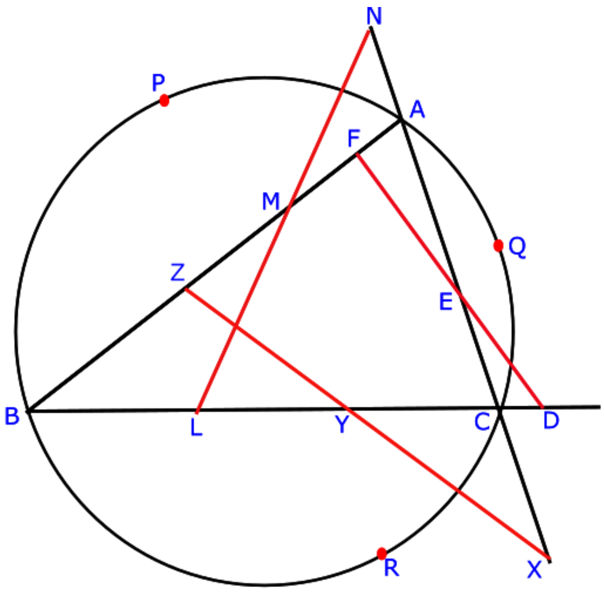
The Utility of the result:

This month's rider is of immense utility to the Geometry lovers. So far, there doesn't seem to be any formula for measuring the length of the Simson line. Here, this result measures the Simson line LMN. ie.

$$LMN = \frac{PC \times AB}{D} = \frac{PF \times AB}{2R} = \boldsymbol{PC \times Sin\ C}$$

$$[\because AB = 2R\ Sin\ C]$$



In the above picture, ΔABC inscribed in the circle. P, Q & R are points on the minor arcs AB, AC and BC respectively. LMN, DEF & XYZ are the Simson lines drawn with respect to the points P, Q & R.

Now, $\boldsymbol{LN = PC \times Sin\ C}$

$\boldsymbol{DF = QB \times Sin\ B}$

$\boldsymbol{XZ = RA \times Sin\ A}$
